ME 305 Fluid Mechanics I

Part 3

Introduction to Fluid Flow

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Field Representation

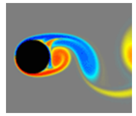
- As a fluid moves, its properties in general change from point to point in space and from time to time.
- In field representation of a flow, fluid and flow properties are given as functions of space coordinates and time.

$$p = p\,(x, y, z, t)\ , \qquad \vec{V} = \vec{V}(x, y, z, t)\ , \qquad etc.$$

- If there is no time dependency in a flow field, it is said to be steady, otherwise it is unsteady.

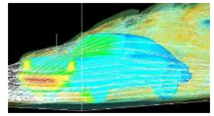
Movie
Steady and unsteady flows

Movie
von Karman vortex street



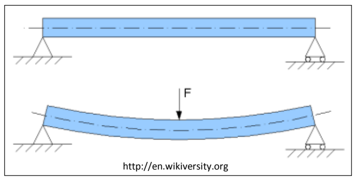
Movie
Wing tip vortices

Movie
Flow around a car



Different Viewpoints for Fluid and Solid Mechanics

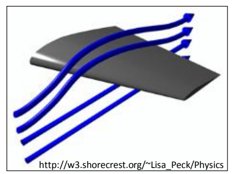
- In solid mechanics we are usually interested in how material moves or deforms. We focus our attention on material and follow its motion/deformation.



- We locate a solid particle (or group of particles) at an initial time and study their motion in time to determine where they go.
- We are interested in particles' trajectories and their final positions, such as golf ball's point of hitting or maximum deflection of the beam's center point.

Different Viewpoints for Fluid and Solid Mechanics (cont'd)

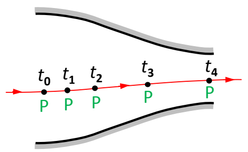
- However, in fluid mechanics we are generally interested in how things behave/change at a point, on a surface or inside a volume. We focus our attention not on material, but on space (location).



- For a lift force generating wing, we need to know the pressure distribution over the wing. We are not really interested in the original locations of fluid particles that cause the lift or where they go after they passed over the wing.
- To measure the amount of liquid flowing in a pipe, we need to make calculations at the exit cross section of it. We do not need to follow the fluid particles that pass through that exit section.

Lagrangian (Material) Description

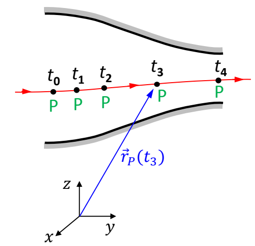
- Identified fluid particles are followed in the course of time as they move in a flow field.
- NOT preferred in fluid mechanics, more suitable to solid mechanics.
- Consider the following experiment where a fluid flows in a converging duct.
- We located a particle P at time t_0 at the entrance of the duct and follow it in time and measure its speed.



Time	Particle P's speed [m/s]
t_0	5
t_1	8
t_2	10
t_3	15
t_4	20

Lagrangian (Material) Description (cont'd)

- In following a particle, the only independent variable is time.
- Space coordinates (x, y, z) of particle P are NOT independent variables.
- When we select a particle by identifying it at its initial location at an initial time, its location at a future time, say t_3, depends on which particle we are following and the value of t_3.



- Properties of particle P are in general expressed as

position of P : $\vec{r}_P(t)$

velocity of P : $\vec{V}_P(t)$

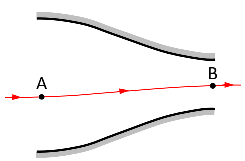
pressure of P : $p_P(t)$

etc.

Lagrangian description

Eulerian (Spatial) Description

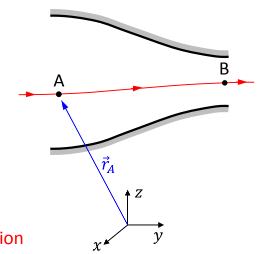
- Attention is focused at fixed points (or area or volume) in the flow field and the variation of properties at these points are determined as fluid particles pass through these points.
- This is the preferred viewpoint for fluid mechanics.
- Consider the same flow in the converging duct, but now concentrating at two points, A and B (or two sections, inlet and exit).



Time	Speed at A	Speed at B
t_0	5	20
t_1	5	20
t_2	5	20
t_3	5	20
t_4	5	20

Eulerian (Spatial) Description (cont'd)

- Now both time and space coordinates are independent variables.
- Location of point A (or B) does NOT depend on the flow field or time.
- Fluid and flow properties at a point (e.g. point A) are expressed as



position : $\vec{r}_A(x_A, y_A, z_A)$

velocity : $\vec{V}_A(x_A, y_A, z_A, t)$

pressure : $p_A(x_A, y_A, z_A, t)$, etc.

Eulerian description

- The duct flow described in the previous slides is said to be steady if the flow properties (such as velocity, pressure, etc.) do not change with time.
- For steady flows time is NOT a variable in the Eulerian description.
- But time is always an independent variable in the Lagrangian description, even for steady flows. Without time, a fluid particle simply can not move.

Lagrangian vs. Eulerian Description

Exercise : Are the following descriptions Lagrangian or Eulerian ?

- A doctor using X-ray opaque dye to trace blood flow in arteries.
- A civil engineer studying the traffic load of a highway by focusing at a certain section of the road and counting the number of cars passing in front of him during a certain period of time.
- A student performing a wind tunnel experiment and measuring the velocity at different points of a flow field by manually moving a velocity measuring probe.

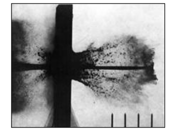
- Fluid dynamic measurements performed in the lab are suited to the Eulerian description. A velocity or pressure probe inserted in a flow field do NOT move with the flow, but provide data at the locations we point it to.

Exercise : Visit Storm Chaser's web site to see the use of a Lagrangian type probe for gathering data inside a twister. Also you can watch the movie Twister to see such probes in action.

Use of Eulerian Description for Solid Mechanics

- Eulerian description is preferred in studying very high deformation solid mechanics problems, in which solids show fluid-like behavior.

Exercise : Do a research on the working principle of ''shaped charge'' used for armor penetration. Watch the movie http://www.youtube.com/watch?v=LudNqf56AFo

Shaped charge and its penetration

http://defense-update.com

Exercise : Watch the following movies in which solids undergo very excessive (fluid-like) deformation.

Aluminum extrusion : http://www.youtube.com/watch?v=9mQ2ic-kDlk

Deep drawing : http://www.youtube.com/watch?v=PBB3utteDq0

Crash test : http://www.youtube.com/watch?v=CcXhjH0hex0

Use of Lagrangian Description for Fluid Mechanics

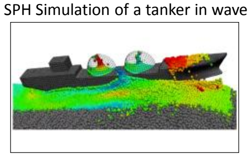
- A doctor using an X-ray opaque dye to trace blood flow in arteries performs a Lagrangian study.
- In some Computational Fluid Dynamics (CFD) studies motion of fluid particles are modeled in a Lagrangian way.

Exercise : Watch this particle simulation of flow around a car

http://www.youtube.com/watch?v=RuZQpWo9Qhs

Exercise : RealFlow is a particle based fluid simulation software used in film-making and television industry. Visit http://www.realflow.com/product/production/casestudies to see its capabilities.

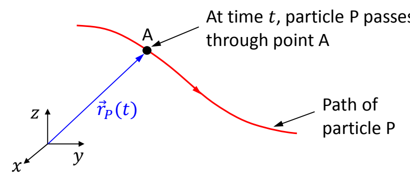
Movie
SPH Simulation of a tanker in wave



Movie
RealFlow Demo Reel

Lagrangian - Eulerian Relation

- Consider a property N (can be velocity, density, pressure, etc.) in a flow field.
- At time t fluid particle P passes through a point A in space.



Exercise: Work on the details of the following important relation

Rate of change of property N of particle P at time t from a Lagrangian point of view = Rate of change of property N at point A from an Eulerian point of view

Lagrangian - Eulerian Relation (cont'd)

$$\boxed{\frac{dN}{dt} = \frac{\partial N}{\partial t} + (\vec{V} \cdot \nabla) N}$$

Material derivative
(Substantial derivative)
(Total derivative)

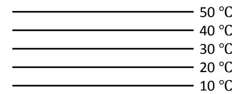
Local derivative
(Partial derivative)

Convective derivative

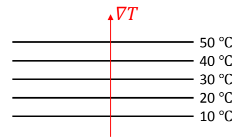
- Material derivative : Rate of change of property N in the material description (following a particle)
- Local derivative : Rate of change of property N (at a fixed point) with time only. For a steady flow this term is zero for any property.
- Convective derivative : Change of property N (at a fixed point) with space only, i.e. at a fixed time. If there is no flow this term is zero.

Understanding the convective derivative $(\vec{V} \cdot \nabla) N$

- Let the property N represents the temperature T.
- Consider the following simple temperature field (shown with constant T lines).

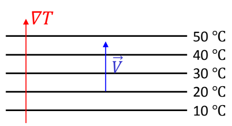


- In this 2D temperature field T changes only in the vertical direction.
- Gradient of temperature (∇T) is in the vertical direction.
- If you move in this direction you'll feel the most rapid change of T.



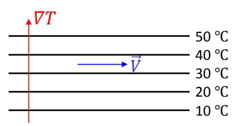
Understanding the convective derivative $(\vec{V} \cdot \nabla) N$

- Consider moving in this T field in different directions.



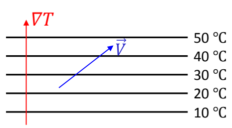
If we move parallel to ∇T we feel the maximum temperature change

$(\vec{V} \cdot \nabla) T = \vec{V} \cdot \nabla T = \text{maximum}$



If we move perpendicular to ∇T we feel no temperature change

$(\vec{V} \cdot \nabla) T = \vec{V} \cdot \nabla T = 0$

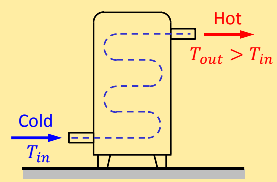


If we move at an angle to ∇T we feel a nonzero temperature change

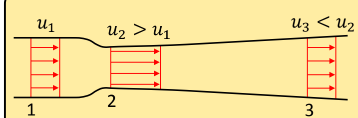
$(\vec{V} \cdot \nabla) T = \vec{V} \cdot \nabla T \neq 0$

Understanding the convective derivative (cont'd)

$$\frac{dN}{dt} = \frac{\partial N}{\partial t} + (\vec{V} \cdot \nabla) N$$



- Steady state operation of a water heater.
- $\partial T/\partial t$ at any point is zero, but dT/dt of a moving fluid particle is not zero.
- Partial derivative of T is zero, but convective derivative is not.



- Steady, uniform flow in a converging-diverging nozzle.
- Fluid particles first accelerate and than decelerate.
- $\partial u/\partial t$ at any point is zero, but du/dt of a moving fluid particle is not.
- Partial derivative of u is zero, but convective derivative is not.

Acceleration of a Fluid Particle

- Selecting N as $\vec{V}$ in equation $\frac{dN}{dt} = \frac{\partial N}{\partial t} + (\vec{V} \cdot \nabla)N$, acceleration of a fluid particle can be obtained as

$$\underbrace{\frac{d\vec{V}}{dt}}_{\text{Acceleration } (\vec{a})} = \underbrace{\frac{\partial \vec{V}}{\partial t}}_{\text{Local acceleration}} + \underbrace{(\vec{V} \cdot \nabla)\vec{V}}_{\text{Convective acceleration}}$$

- Components of the acceleration vector in Cartesian coordinate system are

$$a_x = \frac{du}{dt} = \frac{\partial u}{\partial t} + (\vec{V} \cdot \nabla)u \quad = \quad \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

$$a_y = \frac{dv}{dt} = \frac{\partial v}{\partial t} + (\vec{V} \cdot \nabla)v \quad = \quad \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$$

$$a_z = \frac{dw}{dt} = \frac{\partial w}{\partial t} + (\vec{V} \cdot \nabla)w \quad = \quad \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}$$

Acceleration of a Fluid Particle (cont'd)

Exercise : Using the following ∇ operator in the cylindrical coordinate system

$$\nabla = \frac{\partial}{\partial r}\vec{i_r} + \frac{1}{r}\frac{\partial}{\partial \theta}\vec{i_\theta} + \frac{\partial}{\partial z}\vec{i_z}$$

and the fact that in cylindrical coordinate system unit vectors have the following non-zero derivatives

$$\frac{\partial \vec{i_r}}{\partial \theta} = \vec{i_\theta} \quad \text{and} \quad \frac{\partial \vec{i_\theta}}{\partial \theta} = -\vec{i_r}$$

derive the following acceleration components

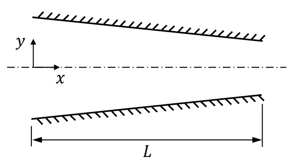
$$a_r = \frac{\partial V_r}{\partial t} + V_r\frac{\partial V_r}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_r}{\partial \theta} + V_z\frac{\partial V_r}{\partial z} - \frac{V_\theta^2}{r}$$

$$a_\theta = \frac{\partial V_\theta}{\partial t} + V_r\frac{\partial V_\theta}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_\theta}{\partial \theta} + V_z\frac{\partial V_\theta}{\partial z} + \frac{V_r V_\theta}{r}$$

$$a_z = \frac{\partial V_z}{\partial t} + V_r\frac{\partial V_z}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_z}{\partial \theta} + V_z\frac{\partial V_z}{\partial z}$$

Acceleration of a Fluid Particle (cont'd)

Exercise : Consider one-dimensional, steady, incompressible flow through a converging channel.



Velocity field is given by

$$u = U_o\left(1 + \frac{x}{L}\right)$$
$$v = 0$$
$$w = 0$$

a) Determine the acceleration field, $a(x)$, by using the Eulerian method.

b) Using the Lagrangian method, determine the equations for the position and acceleration of the fluid particle, which is located at $x = 0$ at time $t = 0$.

c) Show that both expressions for the acceleration give identical results, as the fluid particle exits the channel at $x = L$.

Flow Classification as 1D, 2D and 3D

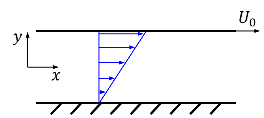
- Following Munson's book (different in Aksel's book)

  Three-dimensional flow : all 3 velocity components are nonzero.

  Two-dimensional flow : only 2 velocity components are nonzero.

  One-dimensional flow : only 1 velocity component is nonzero.

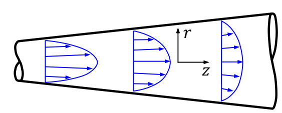
Couette flow



1D flow

$u \neq 0$
$v = 0$
$w = 0$

Axisymmetric nozzle flow

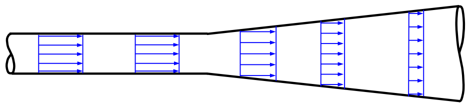


2D flow

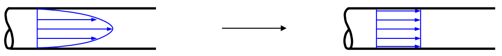
$V_z \neq 0$
$V_r \neq 0$
$V_\theta = 0$

Uniform Flow

- The flow is said to be uniform at a cross section if the only nonzero velocity component is the one perpendicular to the cross section, and the velocity is not changing across the section.



- Uniform flow simplification as used above disregards the no slip condition.
- Instead of the actual velocity profile it uses the average speed at a cross section.



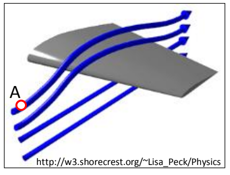
Actual velocity profile — Simplified as uniform

Flow Classification as Steady, Unsteady

- Steady flow : Local derivatives ($\partial\ /\partial t$) are zero in a flow field. Properties at a fixed point do not change in time.
  - See slide 3-16 for two examples.
  - A centrifugal pump working constantly at the same speed between the same input and output conditions is said to be working steadily, although there is a rotating blade inside it.
  - Air flow around a car moving at constant speed is considered to be steady, although there are fluctuations in the wake region behind the car.
- Unsteady flow : Local derivatives (at least for 1 property) are nonzero. Properties at a fixed point change in time.
  - If the inlet water temperature of the heater shown in slide 3-16 changes with time, it will be an unsteady flow.
  - Pulsatile blood flow in our veins is unsteady. But it is a special kind of unsteady flow, it is time periodic. It repeats itself after a certain period.
  - von Karman vortex street of slide 3-2 is also unsteady and time periodic.
  - A gusty wind blowing over a house is unsteady.

Flow Classification as Steady, Unsteady (cont'd)

- Sometimes an unsteady flow can be studied as steady by a proper choice of reference frame.
- Consider the following wing moving at a constant speed in still air.
- For an observer fixed at the ground this flow is unsteady.
- At an upstream point A, initially air speed is zero. But as the wing approaches point A, it will push the air there. Observer fixed at the ground will observe different things at point A at different times.

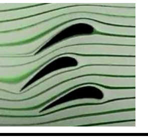


- The same flow becomes steady with respect to an observer moving with the wing. This observer will always see the same air motion around him/her. Nothing will change in time.
- Similar simplifications are observed when turbomachinery flows are studied using a rotating reference frame.

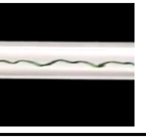
Flow Classification as Laminar, Turbulence

- Laminar flow is a well-ordered state of flow in which adjacent fluid layers move smoothly with respect to each other.
  - Laminar flow is usually associated with low speeds and high viscosities.
- Turbulent flow has random, unsteady, 3D fluctuations. There is intense mixing and rotation.
  - Turbulent flow is usually associated with high speeds.
  - Turbulent flows are, by far, more common than laminar ones.
  - Although a turbulent flow always have unsteadiness in it, it may be steady in the mean (in a time averaged sense). Similarly it can be 2D or 1D in the mean.

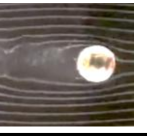
Movie
Laminar flow over cylinders and airfoils



Movie
Laminar to turbulent transition in a pipe



Movie
Wake behind a cylinder

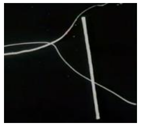


Pathlines, Streaklines and Streamlines

- These are the common ways used to visualize a flow field.
- Pathline is a line traced out by a fluid particle as it flows in a flow field.
  - Pathline is a Lagrangian concept.
  - In laboratory it can be generated by marking (dying) a small fluid element and taking time exposure photograph of its motion.
- Streakline is a line that joins the particles in a flow that have previously passed through a common point.
  - In laboratory it can be generated by continuously injecting dye (or bubbles) at a point and observing the collection of dyed particles as they move in the flow.
- Streamline is a line that is everywhere tangent to the velocity field.
  - It is a mathematical tool, rather than an experimental technique.

- For a steady flow all these three are the same.
- For an unsteady flow they are all different.

NCFMF Movie
Pathline, streakline and streamline comparison for unsteady oscillating plate flow

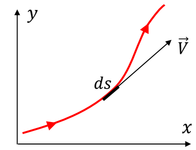


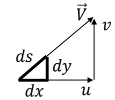
Equation of a Streamline

- Consider a streamline in a 2D flow field.
- At any point velocity vector is tangent to it.
- Slope of the line at any point (dy/dx) should be equal to the velocity component ratio (v/u)

$$\frac{dy}{dx} = \frac{v}{u}$$

which can also be written as $\frac{dx}{u} = \frac{dy}{v}$





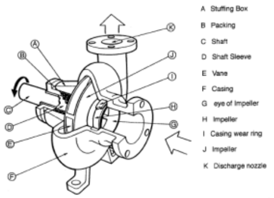
- This can be generalized to a 3D flow as $\boxed{\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}}$

- If the velocity field is known as a function of x, y and z (and t if the flow is unsteady), the above equation can be integrated to give the equation of streamlines.

Exercise : For the velocity field given by $\vec{V} = 2x\vec{\imath} - 2y\vec{\jmath}$, determine the equation of the streamline that passes through point P(2,2,0).

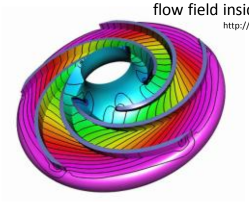
Differential vs. Integral Formulation

- Differential formulation provides a very detailed solution of a flow field.
- When used with Eulerian point of view, it provides information at all points in the problem region at all times of interest.
- It requires the solution of differential equations for conservation laws (mass, linear momentum and energy).
- Analytical solution of conservation equations are available only for a few very simple problems. Computational Fluid Dynamics (CFD) provides an alternative.



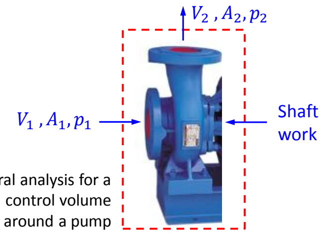
Parts of a centrifugal pump
http://www.mcnallyinstitute.com

Detailed solution of the flow field inside a pump
http://www.grc.nasa.gov



Differential vs. Integral Formulation (cont'd)

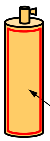
- Integral formulation used with Eulerian viewpoint focuses at a fixed region of space (control volume).
- It studies the interaction of this control volume with its surroundings.
- It is used to determine gross flow effects (not details), such as the lift force generated by a wing , thrust generated by a jet engine or the shaft work required to run a pump.
- It provides less information compared to differential approach. But it has much simpler mathematics. No differential equation is solved.



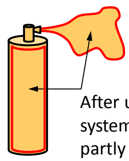
Integral analysis for a control volume around a pump

Closed System vs. Control Volume

- A closed system (or just system) is a fixed, identifiable quantity of mass.
- It can change its position and shape, but it always contains the same fluid particles.
- It is separated from the surroundings by the system boundaries, which is closed to mass transfer. Fluid particles can not pass through it.
- It is closely linked to the Lagrangian description.
- It has the advantage that basic laws (conservation of mass, momentum, energy) can be written for it in a very natural and simple way.



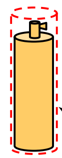
Closed system for the perfume is initially inside the spray can



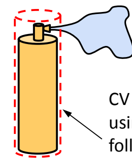
After using it, the closed system is partly inside and partly outside. It follows perfume's motion.

Closed System vs. Control Volume (cont'd)

- A control volume (CV) is a fixed region of a flow field.
- It can NOT change its position or shape, but it contains different fluid particles at different times (Note: Moving/deforming CVs can also be defined).
- It is separated from the surroundings by the control surface (CS), which is open to mass transfer. Fluid particles can pass through the CS.
- It is closely linked to the Eulerian viewpoint.
- Reynolds Transport Theorem (RTT) is used to convert basic conservation laws written for a closed system to equations that can be used for a CV.



Initial shape of the CV.



CV does not change shape after using the perfume. It does not follow perfume's motion. It has an exit, through which perfume leaves.

Basic Laws Written for a System

Conservation of Mass : Mass of a closed system does not change, i.e. time rate of change of a closed system's mass is zero.

$$\frac{dm_{sys}}{dt} = 0 \qquad \text{where} \qquad m_{sys} = \int_{\forall_{sys}} \rho \, d\forall$$

Conservation of Linear Momentum (Newton's 2nd Law) : Sum of all external forces acting on a system is equal to the time rate of change of its linear momentum.

$$\sum \vec{F} = \frac{d\vec{P}_{sys}}{dt} \qquad \text{where} \qquad \vec{P}_{sys} = \int_{\forall_{sys}} \rho \, \vec{V} \, d\forall$$

Conservation of Angular Momentum : Sum of all external torques acting on a system is equal to the time rate of change of its angular momentum.

$$\sum \vec{T} = \frac{d\vec{H}_{sys}}{dt} \qquad \text{where} \qquad \vec{H}_{sys} = \int_{\forall_{sys}} \rho(\vec{r} \times \vec{V}) d\forall$$

Basic Laws Written for a System (cont'd)

Conservation of Energy (1st Law of Thermodynamics) : Energy of a closed system changes by heat and work interaction with its surroundings as follows

Dot means "time rate of change of ". Same as $\frac{d}{dt}$

$$\dot{Q} + \dot{W} = \frac{dE_{sys}}{dt} \qquad \text{where} \qquad E_{sys} = \int_{\forall_{sys}} \rho \, e \, d\forall$$

$$e = u + \frac{V^2}{2} + gz$$

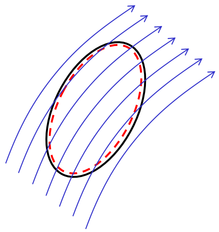
Total energy per unit mass | Internal energy per unit mass | Kinetic energy per unit mass | Potential energy per unit mass

$\dot{Q}$: rate of heat transfer (heat coming into the system is positive)

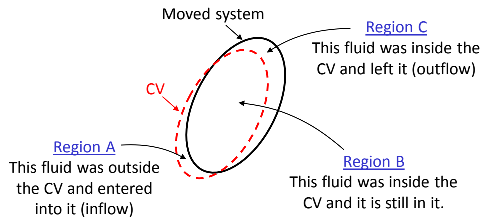
$\dot{W}$: rate of work done (work done on the system is positive)

Reynolds Transport Theorem (RTT)

- All basic laws are naturally written in very simple forms for a closed system.
- But we want to use CVs to study fluid mechanics problems.
- RTT is a general relation between the rate of change of a fluid property in a closed system and the corresponding CV.



At time t, CV and the system coincide



At time $t + \Delta t$, CV stays at its original place but the system moves/deforms with the flow

Reynolds Transport Theorem (cont'd)

- We are interested in the change of an extensive property N such as m, $\vec{P}$, $\vec{H}$ or E.
- The corresponding intensive (per mass) property is η, such as 1, $\vec{V}$, $(\vec{r} \times \vec{V})$ or e.

$$N_{sys} = \int_{sys} \rho\, \eta\, d\forall$$

- From time t to $t + \Delta t$, N_{sys} may change

$$\frac{dN_{sys}}{dt} = \frac{d}{dt}\int_{sys} \rho\eta d\forall = \lim_{\Delta t \to 0} \frac{1}{\Delta t}\left(\int_{sys} (\rho\eta d\forall)_{t+\Delta t} - \int_{sys} (\rho\eta d\forall)_t\right) \qquad \text{Eqn } (*)$$

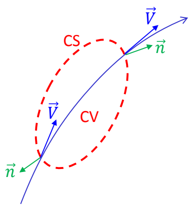
$$\int_{sys} (\rho\eta d\forall)_{t+\Delta t} = \int_{CV} (\rho\eta d\forall)_{t+\Delta t} + \underbrace{\int_{\forall_C} (\rho\eta d\forall)_{t+\Delta t}}_{N \text{ leaving the CV through the exit}} - \underbrace{\int_{\forall_A} (\rho\eta d\forall)_{t+\Delta t}}_{N \text{ entering the CV through the inlet}}$$

$$\int_{sys} (\rho\eta d\forall)_t = \int_{CV} (\rho\eta d\forall)_t$$

Reynolds Transport Theorem (cont'd)

$$\frac{dN_{sys}}{dt} = \lim_{\Delta t \to 0} \frac{1}{\Delta t}\left(\int_{CV} (\rho\eta d\forall)_{t+\Delta t} - \int_{CV} (\rho\eta d\forall)_t + \int_{\forall_C} (\rho\eta d\forall)_{t+\Delta t} - \int_{\forall_A} (\rho\eta d\forall)_{t+\Delta t}\right)$$

$$\frac{\partial}{\partial t}\int_{CV} \rho\eta d\forall \qquad \underbrace{\int_{A_C} \rho\eta(\vec{V}\cdot\vec{n})dA \quad - \int_{A_A} \rho\eta(\vec{V}\cdot\vec{n})dA}_{\int_{CS} \rho\eta\,(\vec{V}\cdot\vec{n})\,dA}$$



$\vec{V}$ is the velocity of the fluid at the control surface CS

$\vec{n}$ is the unit outward normal of the CS.

Exercise : Study the detailed derivation of RTT from a textbook.

Reynolds Transport Theorem (cont'd)

$$\text{RTT:} \quad \underbrace{\frac{dN_{sys}}{dt} = \frac{d}{dt}\int_{sys} \rho\eta d\forall}_{\text{Lagrangian part}} = \underbrace{\frac{\partial}{\partial t}\int_{CV} \rho\eta d\forall + \int_{CS} \rho\eta(\vec{V}\cdot\vec{n})dA}_{\text{Eulerian part}}$$

Rate of change of property N within the system. = Rate of change of property N in the corresponding CV (zero for steady flows) + Net flowrate of property N across the CS (positive for outflow, negative for inflow)

- Important: Left hand side is NOT calculated directly, instead we use

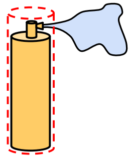
Zero	:	for mass conservation
$\sum \vec{F}$	:	for linear momentum conservation
$\sum \vec{T}$	:	for angular momentum conservation
$\dot{Q} + \dot{W}$	:	for energy conservation

Reynolds Transport Theorem (cont'd)

Example : Let's use RTT to study mass conservation in the spray can.

- For mass conservation, $N = m$ and $\eta = 1$

$$\frac{dm_{sys}}{dt} = \frac{\partial}{\partial t}\int_{CV} \rho \, d\forall + \int_{CS} \rho \, (\vec{V} \cdot \vec{n}) dA$$



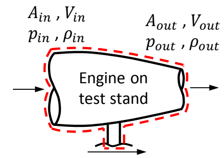
Zero = Time rate of change of perfume's mass inside the spray can. A negative value. + Amount of mass that leaves the can in unit time. A positive value (Nonzero only at the little opening that the perfume can escape from)

Reynolds Transport Theorem (cont'd)

Example : Let's use RTT to study the thrust generated by a jet engine.

- For linear momentum conservation, $N = \vec{P}$ and $\eta = \vec{V}$

$$\frac{d\vec{P}_{sys}}{dt} = \frac{\partial}{\partial t}\int_{CV} \rho\vec{V} d\forall + \int_{CS} \rho\vec{V} \, (\vec{V} \cdot \vec{n}) \, dA$$



Force to hold the engine in place = Thrust force

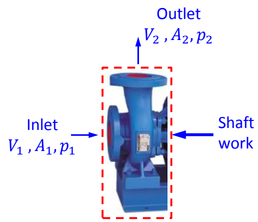
$\sum \vec{F}$ Sum of external forces acting on the CV = Zero for the steady operation of the engine. + Net rate of momentum outflow (Nonzero only at the inlet and exit of the engine)

Reynolds Transport Theorem (cont'd)

Example : Let's use RTT to study the power necessary to run a pump.

- For energy conservation, $N = E$ and $\eta = e$

$$\frac{dE_{sys}}{dt} = \frac{\partial}{\partial t}\int_{CV} \rho e d\forall + \int_{CS} \rho e \, (\vec{V} \cdot \vec{n}) \, dA$$



$\dot{Q} + \dot{W}$ Rate of heat transfer and work done on the CV = Zero for the steady operation of the pump. + Net rate of energy flow through the CS (Nonzero only at the inlet and outlet of the pump)